

An Interval-Based Semantics for Degree Questions: Negative Islands and their Obviation

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Introduction

The goal of this paper is to argue for a new approach to the semantics of degree questions. According to the standard analysis (see, among others, Rullmann 1995 and Beck & Rullmann 1997), a degree question's LF contains a variable that ranges over individual *degrees* and is bound by the degree-question operator *how*. In contrast with this, we will claim that the variable which is bound by the degree-question operator *how* does not range over individual degrees but over *intervals* of degrees, by analogy with Schwarzschild & Wilkinson's (2002) proposal regarding the semantics of comparative clauses.

In other words, while in the standard view, a sentence such as (1) corresponds to the (informal) logical form given in (2-a), we will assume that the right representation is the one given informally in (2-b):

- (1) How fast is Jack driving ?
- (2) a. Standard View : for what degrees *d* of speed, is Jack driving *d*-fast?
b. Interval-Based Semantics: for what intervals *I* of degrees of speed, is Jack driving at a speed included in *I*?

As we'll argue, not only does the interval-based semantics predict the existence of certain readings that are not predicted under the standard view, it is also able (and this will be our main focus), together with other natural assumptions, to account for the sensitivity of degree questions to negative-islands, illustrated by (3) (cf. Rizzi 1990, Szabolcsi & Zwart 1993, Rullmann 1995, among others), as well as for the fact, uncovered by Fox & Hackl (2007), that negative islands can be obviated by some properly placed modals, as in (4) (we call this phenomenon *modal obviation*).

- (3) a. How fast did Jack drive?
b. *How fast didn't Jack drive?
c. *How fast are we allowed not to drive?
- (4) a. How fast are we not allowed to drive?
b. How fast are we required not to drive?

The relevant generalization, which is due to Fox & Hackl (2007), is the following:

Fox and Hackl's generalization:

Negative islands get obviated if negation immediately scopes either just above a possibility modal (as in (4-a)) or just below a necessity modal (as in (4-b)).

We will show that this generalization follows from the combination of the interval-based semantics and a principle, due to Dayal (1996), according to which any question presupposes that it has a maximally informative answer, i.e. a true answer that entails all the other true answers. Fox & Hackl's (2007)

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account of these facts, which we will present in the next section, also resorts to this principle, and in this respect our proposal is a direct heir to theirs; we will see, however, that our proposal allows us to dispense with two others of Fox & Hackl's (2007) assumptions, namely the hypothesis that every measurement scale is *dense* (see next section for a definition), and more importantly, the view that Dayal's condition is computed in a way that is blind to contextual information (see subsection 1.3)

1. Background: Fox & Hackl (2007)

1.1. The ingredients of Fox & Hackl's (2007) account

Fox & Hackl (2007) offers an explanation for the paradigm in (3) and (4) based on the following set of assumptions:

- Degree-predicates such as *fast* or *tall* denote a relation between individuals and *degrees*, and are governed by the following meaning postulate, which makes them *monotone decreasing* with respect to their degree argument:

$$\forall x \forall d \forall d' (d' \leq d \rightarrow (\text{tall}(x)(d) \rightarrow \text{tall}(x)(d')))$$

Informally speaking, such a meaning postulate ensures that being *d*-tall is equivalent to being *d*-tall or more.

- The degree question operator *how* is an operator that binds a variable ranging over degrees (in the scale determined by the adjective that *how* combines with):

(5) Logical Form of *how tall is John?*:
How_d [John is *d*-tall]?

- The universal density of measurement: all scales are *dense*, i.e. for any two degrees *d*₁ and *d*₂ in a given scale, there is a degree *d*₃ between *d*₁ and *d*₂:

$$\forall d_1 \forall d_2 ((d_1 < d_2) \rightarrow (\exists d_3 d_1 < d_3 < d_2))$$

- Maximal Informativity Principle (following Dayal 1996): any question presupposes that it has a maximally informative answer, i.e. a true answer which logically entails all the other true answers. We call the presupposition induced by the Maximal Informativity Principle *Dayal's presupposition* (see also Beck & Rullmann 1999).

1.2. Fox & Hackl's (2007) account of negative islands and of modal obviation

1.2.1. Simple negative islands

Given the above assumptions, the logical form for a question like (3-b) is the one given in (6), and Dayal's presupposition amounts to the claim that among all the true sentences of the form *Jack did not drive d-fast or more than d-fast*, there is one that entails all the others.

(6) How_d [Jack did not drive *d*-fast]?

Note that for any *d*, *d'*, with $d \leq d'$, the proposition that Jack didn't drive *d*-fast entails the proposition that Jack didn't drive *d'* fast (for if $d \leq d'$, then the proposition that Jack's speed is at most *d* entails the proposition that Jack's speed is at most *d'*). So the maximally informative true answer, if it exists, must be based on the *smallest* degree *d* such that Jack's speed was not *d* or more than *d*.

Now suppose that Jack's exact speed was 50mph. Then for any $d > 50$, Jack did not drive *d*-fast; but is there a *smallest* *d* such that Jack did not drive *d*-fast, i.e. a smallest *d* above 50? Assuming that the scale of speeds is dense, there cannot be such a degree: for any degree $50 + \epsilon$, however small ϵ is, there is another degree $50 + \epsilon'$ strictly between 50 and $50 + \epsilon$ (take ϵ' smaller than ϵ).

Therefore the presupposition that there is a maximally informative true answer can never be met.

1.2.2. Modal obviation

The acceptability of the questions in (4) is accounted for as follows. (4-a)'s logical form is (informally) the following:

- (7) For what degree d of speed, we are not allowed to drive d -fast?

Now suppose that the law states that we are not allowed to drive at 65mph or faster, and says nothing more. Then for any speed d strictly smaller than 65mph, it is false that we are not allowed to drive d -fast. Hence 65mph is the smallest speed d such that we are not allowed to drive d -fast or more; furthermore, not being allowed to drive at 65mph or more entails not being allowed to drive at any higher speed. Hence the proposition that we are not allowed to drive at 65mph or more is the most informative true proposition of the form *we are not allowed to drive d -fast*, and Dayal's condition is met. In other words, the condition that there be a maximally informative answer can be met in the case of (4), which is why such a question is acceptable.

This reasoning extends trivially to (4-b), simply because *being required not to do X* is equivalent to *not being allowed to do X* . Fox & Hackl (2007) also show that in cases where negation *follows* a possibility modal, as in (3-c), or precedes a necessity modal, the corresponding questions are predicted to be unacceptable.

1.3. Apparently discrete scales, modularity, and blindness to contextual information

This importance of the assumption that all scales are dense is particularly clear when one turns to scales that are not intuitively dense. Consider for instance the following contrast:

- (8) a. How many children does John have?
b. *How many children doesn't John have?

The unacceptability of (8-b) is explained in exactly the same way as that of (3-b). Suppose John has exactly three children. Then John, for sure, does not have four children; but it is also true that he does not have 3.5 children, or 3.05 children, or 3.005 children, and so on ... Hence there does not exist a minimal number n such that Jack does not have n children or more, and therefore (8-b) can not have a maximally informative answer. But if the degree variable in (8-b) could be restricted to range over natural numbers, there would actually exist a minimal degree n such that Jack does not have at least n children - namely, take $n = 4$. But the universal density of measurement scales entails that there is no scale consisting of just the natural numbers.

Such a view can be defended on the ground that the knowledge that the number of children someone has is an integer is not purely linguistic or "logical"; rather, it is a form of lexical or encyclopedic knowledge, and therefore there is no reason why *how many* should be constrained to bind a variable whose range of values includes only the natural numbers. Yet in normal contexts, the *relevant* values for the variable bound by *how many* in (8-b) consist only of the integers; if it were possible to restrict the range of the numerical variable to a salient domain of quantification (such as the natural numbers), the condition that there be a maximally informative answer would actually be met for (8-b). So Fox and Hackl do not only need to claim that all scales are dense; they also need to assume a modular system in which some semantic and pragmatic processes operate in isolation and are blind to contextual information, in particular to possible contextual restrictions on the range of variables.

Fox & Hackl's (2007) explicit goal is to challenge certain widely accepted assumptions regarding the relationship between grammar, pragmatic processes, lexical meaning and contextual factors. We consider it worthwhile to investigate an alternative account of the negative-island facts which can dispense with the hypothesis that grammar always treats all scales as dense.¹

¹Fox & Hackl (2007) show that the universal density of measurement scales is able to account for many other interesting facts outside the realm of degree questions, which we do not address in this paper.

2. The Interval-Based Account

In this section, we will present a first version of our proposal (subsection 2.1), before showing how it accounts for the basic paradigm described in (3) and (4) (subsection 2.2). Then we will provide some independent motivation for the proposal (subsection 2.3). In the subsequent and last section (section 3), additional data will lead us to enrich the basic proposal.

2.1. The proposal

Our proposal is based on the following two ingredients.

- Instead of treating scalar predicates as denoting a relation between individuals and degrees, we assume that they denote a relation between individuals and *intervals* of degrees² (cf. Schwarzschild & Wilkinson 2002):

(9) $\|\text{tall}\| = \lambda D_{\langle d,t \rangle} : D \text{ is an interval. } \lambda x.x\text{'s height} \in D$ (where *interval* is defined as in (10))

(10) Given a scale E , i.e. an ordered set (E, \leq) , an *interval* on E is a subset D of E such that:
 $\forall d_1, d_2, d_3, (d_1 \in D \wedge d_3 \in D \wedge d_1 \leq d_2 \leq d_3) \rightarrow d_2 \in D$

Note that the notion of interval is well defined for discrete scales as well as for dense scales.

As a result, the LF of a *how tall* question will be (11-a), which can be informally paraphrased as in (11-b):

- (11) a. $\text{How}_{D_{\langle d,t \rangle}} [\dots D\text{-tall} \dots]$
b. For what intervals D of degrees of heights, is it the case that $[\dots D\text{-tall} \dots]$?

- Like Fox & Hackl (2007), we adopt a version of Dayal's (1996) *Maximal Informativity Principle*, with a slight modification:³ we require that the maximally informative answer whose existence is presupposed be *contextually informative* in the sense that it is not entailed by the common ground.

Maximal Informativity Principle: A question presupposes that it has a maximally informative answer, where a maximally informative answer is defined as a *true answer that is not entailed by the common ground and entails all the true answers*.⁴ As before, we call this presupposition *Dayal's presupposition*.

We assume that a question that can never have a maximally informative answer, i.e. for which Dayal's presupposition is a logical contradiction, is unacceptable (as in Fox & Hackl 2007 and Abrusán 2007)

2.2. Explaining the basic paradigm

2.2.1. Simple degree questions

A simple degree question such as (12-a) receives the interpretation given in (12-b).

- (12) a. How tall is Mary?
b. For what interval I , does Mary's height belong to I ?

²See however footnote 6.

³The rationale for this change will be apparent in subsection 2.2.2.

⁴We use the term *answer* in a narrow, technical sense: an *answer* to a degree-question of the form $[\text{How}_D \phi(D)]$ is a proposition that belongs to the Hamblin-set of the question, i.e. a proposition that can be expressed as $\phi(D)$, for some interval D . Crucially, many propositions that can intuitively serve as answers are not in this set. As far as we know, every semantic account of weak islands, starting with Szabolcsi & Zwart (1993), needs to make very specific assumptions about the structure of the set of possible answers to a given question. See the appendix to Abrusán & Spector (2008) for a short comparison of our approach with that of Szabolcsi & Zwart (1993).

Now let h be Mary's height. Clearly, any answer based on an interval that includes h is a true answer; furthermore, the proposition that Mary's height belongs to a given interval I_1 entails the proposition that Mary's height belongs to I_2 , for any I_2 that includes I_1 . Consequently, the proposition that Mary's height belongs to the interval $[h, h]$ (i.e. *is h*) expresses a true answer that entails all the other true answers, hence is the maximally informative answer. Hence in such a basic case, Dayal's presupposition is always met, and a fully cooperative and informed speaker is expected to use this maximally informative answer, i.e. to give Mary's exact height as an answer - clearly a correct prediction.

2.2.2. Predicting Negative Islands

We now turn to negative islands. According to the interval-based semantics, (13-a) is interpreted as (13-b).

- (13) a. # How fast didn't Mary drive?
 b. For what interval I , Mary's speed was not in I

The unacceptability of (13) is derived in two steps:

- First, let us show that (13) has a true answer that entails all the true answers if and only if Mary's speed was 0. Let s be Mary's speed. Let us assume that s is distinct from 0. Then the set of intervals such that s is not in them consists of a) all the intervals strictly above s , and b), all the intervals strictly below s . Hence the set of all the true answers to (13) is the set of answers based on such intervals (where an answer is said to be 'based' on an interval I if it expresses the proposition that Mary's speed was not in I). Now note that any answer based on an interval above s fails to entail an answer based on an interval strictly below s , and vice versa. Therefore there cannot be a true answer to (13) that entails all the other true answers. To sum up, if s is distinct from 0, there cannot be a maximally informative answer. If $s = 0$, then there is a true answer that entails all the true answers, namely, the proposition that states that Mary's speed was not in the interval $]0, +\infty[$, which is equivalent to the claim that $s = 0$. So (13) has a true answer that entails all the true answers if and only if Mary's speed was 0.
- Second, let us temporarily assume that Dayal's presupposition is not contradictory. Dayal's presupposition entails that there is a true answer that entails all the true answers, i.e., given what has just been shown, that Mary's speed was 0. This means that (13) can be felicitous only when it is common knowledge that Mary's speed is 0, or, equivalently, that her speed is not included in $]0, +\infty[$. But then the maximally informative answer, namely the proposition that Mary's speed is not included in $]0, +\infty[$, is in fact already entailed by the common ground. Since a maximally informative answer must not only be a true answer that entails all the true answer, but must also be contextually informative, this proposition is not a maximally informative answer after all.

It follows that there cannot be a maximally informative answer, i.e. that Dayal's presupposition is contradictory. Note that the above proof does not make reference to whether the underlying scale is dense or discrete, a point that we will emphasize in subsection 2.2.5.

2.2.3. Accounting for Fox & Hackl's (2007) obviation facts

Recall that a possibility modal scoping below negation is able to obviate negative islands, as was illustrated by example (4-a), repeated here as (14):

- (14) How fast are we not allowed to drive?

According to the interval based account, (14) receives the following interpretation:

- (15) For what interval I , it is not allowed that our speed be in I ?

Now suppose that the law determines a maximal permitted speed and does nothing more. Call this speed limit l . Then the set of all the intervals I such that it is not allowed that our speed be in I is the set of all the intervals that are strictly above l . Consider in particular the answer to (14) based on the interval

$]l, +\infty[$. Clearly, this answer is true and entails all the other true answers (since the information that our speed must not be in $]l, +\infty[$ tells us everything there is to know in this context). Hence in such a context Dayal's presupposition is met, and therefore negation is not predicted to trigger a weak-island effect.

This reasoning extends straightforwardly to the case where negation scopes just below a necessity modal, as in (4-b), because being required not to drive at a speed contained in a given interval I is equivalent to not being allowed to drive at a speed contained in I .

2.2.4. No obviation if the possibility modal scopes above negation

When the possibility modal takes scope not below but above negation, then the question is not acceptable, as was shown in (3-c), which we repeat below in (16-a), and whose interpretation according to the interval-based semantics is given in (16-b):

- (16) a. # How fast are we allowed not to drive?
 b. For what interval I , it is allowed that our speed not be in I ?

To show how this prediction comes about, let us first point out the following logical fact:

- (17) For any two intervals I_1 and I_2 , the two following statements are equivalent:
 a. *It is not allowed that our speed be in I_1* entails *It is not allowed that our speed be in I_2*
 b. I_2 is included in I_1

From the observation in (17), it follows that the maximally informative true answer to (16), if it exists, is based on an interval that includes all the intervals that yield a true answer. Let us call this interval the 'maximally informative interval'. Let us then consider several possible cases:

- First case. There is no particular speed s such that our speed must be exactly s .
 In this case, for any speed d , we are allowed not to drive at speed d , i.e. it is allowed that our speed be in the (degenerate) interval $[d,d]$. But given that the maximally informative interval M , if it exists, must include all the intervals I such that it is allowed that our speed be not in I , M must in fact be $[0, +\infty[$. But this would mean that it is allowed that our speed not be in $[0, +\infty[$, i.e. that there is a permissible world in which we have no speed (not even the null speed), which is obviously contradictory. Hence there cannot be a maximally informative interval.
- Second case. There is a particular speed s , distinct from 0, such that our speed must be exactly s .
 Since $s \neq 0$, it is allowed that our speed not be 0, and it is also allowed that our speed not be d , for any d strictly above s . Hence the maximally informative interval, if it exists, must include both 0 and any speed above s , hence must again be $[0, +\infty[$, which, as explained above, is contradictory.
- Remaining case. Our speed must be exactly 0.
 It follows from the two previous reasonings that there can be a true answer that entails all the true answers only if our speed must be exactly 0. In such a case there is a true answer to (16) that entails all the true answers, namely the proposition according to which we are allowed not to drive at a speed included in $]0, +\infty[$.

We can now conclude that there cannot be a maximally informative answer in our sense, i.e. a true answer that entails all the true answers *and is contextually informative, namely is not entailed by the common ground*. For it follows from the above reasoning that Dayal's presupposition entails that our speed must be 0. Hence the only contexts where it could be satisfied are those where it is common knowledge that our speed must be 0. But in such contexts, all the true answers to (16) are already common knowledge, and so no true answer is contextually informative.

2.2.5. Discrete scales

Our proposal does not rely on any particular assumption about the precise structure of scales, besides the fact that they are totally ordered sets. So the account works equally well for dense and discrete scales.

Let us have a brief look at negative *how many*-questions.⁵

- (18) a. # How many children doesn't Jack have?
b. For what interval I of integers, the number of children that Jack has is not in I?

The explanation of the deviance of (18-a) is exactly the same as in the case of (13-a). Namely, if Jack has, say, exactly n children, then for any interval D either strictly above or strictly below n , it is true that the number of children that Jack has is not in D . But for any D_1 strictly above n and any D_2 strictly below n , neither the answer based on D_1 entails the one based on D_2 , nor does the latter entail the former. Therefore the only case where there could be a maximally informative answer is when $n = 0$. Hence Dayal's presupposition entails that Jack has no children, but, again, any context which could satisfy it is a context in which there is no contextually informative true answer. Hence there cannot be a maximally informative answer.

2.3. *The interval-based reading exists*

Having shown that the combination of the interval-based semantics and the principle of maximal informativeness account for Fox and Hackl's generalization, it is also necessary to show that such a semantics is reasonable, i.e. that the readings it predicts for degree-questions exist. Thus consider the following question:

- (19) How fast are we required to drive on this highway?

Suppose that in actual fact, we are required to drive at a speed between 45mph and 75mph on the relevant highway. According to the 'standard' view of degree-questions, the complete answer to (19) should be the most informative proposition of the form *we are required to drive at X mph at least*, i.e., in this situation *We are required to drive at at least 45mph*. According to the interval-based semantics, the complete answer in this situation is strictly stronger, as it is the proposition that states that we are required to drive between 45mph and 75mph. Most informants agree that both answers could be given by a fully informed and cooperative speaker. This suggests that both the readings predicted by the standard account and by the interval-based semantics exist. On the one hand, the existence of the second reading provides support for our proposal; on the other hand, we do not account for the possibility of another reading corresponding to what is predicted by the 'standard' approach. In the appendix to Abrusán & Spector (2008) and in work in progress, we present a more sophisticated version of our analysis that is able to predict both readings while still accounting for the facts uncovered by Fox & Hackl (2007).⁶ For the time being, our aim is to establish the existence of the interval-based reading.

We can confirm the existence of the interval-based reading by paying attention to the interpretation of *embedded* degree questions. Thus consider the interpretation of (20-b) below, uttered just after the discourse given in (20-a):

- (20) a. Jack and Peter are devising the perfect Republic. They argue about speed limits on highways. Jack believes that people should be required to drive at a speed between 50mph and 70mph. Peter believes that they should be required to drive at a speed between 50mph and 80mph. Therefore . . .
b. Jack and Peter do not agree on how fast people should be required to drive on highways.

The point is that (20-b) can clearly be judged true in the context given in (20-a). Let us assume (following for instance remarks by Sharvit 2002) that for X and Y to disagree on a given question Q ,

⁵As is well known, negative islands in degree questions arise only for one type of reading, sometimes called the *amount*-reading, which is usually analyzed as involving reconstruction of the numerical variable; see Heycock (1995) and Fox (1999) and the references cited therein.

⁶In this more sophisticated proposal, we actually revert to the view that scalar adjectives denote a relation between individuals and degrees, and we derive the kind of denotation shown in (9) – according to which a scalar adjective denotes a relation between individuals and intervals – by applying an operator, noted **II**, to the basic denotation of an adjective, following suggestions by Schwarzschild (2004) and Heim (2006). Several distinct readings are then predicted for certain degree-questions, depending on the scope of the operator **II**.

there must be at least one potential answer A to Q such that X and Y do not assign the same truth-value to A (in the technical and narrow sense of the term ‘answer’, cf. footnote 4). Then for Jack and Peter to disagree on how fast people should be required to drive, i.e. for (20-b) to be true, there must be at least one answer to *How fast should people be required to drive on highways?* about which Jack and Peter disagree. But note that in the context described in (20-a), Jack and Peter do not actually disagree about the minimal permitted speed; in other words, they agree on the truth value of every proposition of the form *People should be required to drive at least d-fast*. So according to the ‘standard’ view of degree questions, (20-b) is false in such a context (since according to this view, the potential answers to the embedded degree questions in (20-b) are precisely the propositions of the form *People should be required to drive at least d-fast*, about which Jack and Peter have no disagreement). Whether or not there is a reading according to which (20-b) is false given (20-a), it is clearly possible to interpret it as true in this scenario. Therefore the standard view is insufficient. Furthermore, the interval-based analysis straightforwardly accounts for this truth-value judgement. According to the interval-based semantics, (20-b) means that for at least one interval of speeds I, Jack and Peter do not agree on the truth-value of *People should be required to drive at a speed included in I*. And this is clearly the case in the scenario given in (20-a) - namely, Jack believes that people should be required to drive at a speed contained in $I = [50, 70]$, while Peter thinks this is not so.

3. The overgeneration problem

In this section, we will point out cases where our proposal, in its current form, predicts a degree question to be felicitous in a context in which it is not in fact felicitous. We will then enrich the proposal in order to solve this overgeneration problem.

3.1. Problems

Problem #1. Consider the following degree question:

(21) How fast are we required to drive on the highway?

In a context in which it is known that a) there is a maximal permitted speed l and b) there is no obligatory minimal speed, the interval-based semantics predicts (21) to be felicitous: clearly the proposition according to which we are required to drive at a speed included in $[0, l]$ is the maximally informative answer, and therefore the maximal informativity principle is satisfied. Yet clearly (21) is infelicitous in such a context, since it in fact suggests that there is an obligatory minimal speed.

Problem #2. Consider again a paradigmatic case of modal obviation:

(22) How fast are we not allowed to drive on the highway?

In subsection 2.2.3, we have shown that our proposal predicts (22) to be felicitous in a context where it is known that there is a maximal permitted speed. But it turns out that (22) is also predicted to be felicitous in a context where it is known that there is a minimal permitted speed and no maximal permitted speed. Suppose indeed that we are required to drive above a given speed l but we are allowed to drive as fast as we want. Then clearly the most informative answer to (22), according to the interval-based semantics, is simply the proposition that states that we are not allowed to drive at a speed included in $[0, l]$, and so the maximal informativity principle is again satisfied. But this does not correspond to our intuitions, since (22) clearly suggests that there is a maximal permitted speed.⁷

⁷Note that in languages like French, in which degree questions are introduced by expressions such as *At what speed*, the problems discussed in this section do not arise, because the relevant questions are felicitous even in the contexts that were shown to be problematic in the previous subsection:

- (i) a. A quelle vitesse doit-on conduire sur l’autoroute? (*At what speed must one drive on the highway?*)
- b. A quelle vitesse n’a-t-on pas le droit de conduire sur l’autoroute? (*At what speed does one not have the right to drive on the highway?*)

How fast vs. How slow. In all these cases, replacing *fast* with *slow* eliminates the inferences we have just mentioned and gives rise to new ones. For instance, while (21) suggests that there is a minimal permitted speed, a question such as *How slow are we required to drive on the highway* suggests instead that there is a *maximal* permitted speed. So far our proposal is unable to predict this difference.

3.2. A presuppositional solution

3.2.1. Proposal

We now represent the logical form of a degree question as follows:

- (23) $\text{How}_{D\{S, \leq\}} \phi(D)$
 (where D is a variable ranging over intervals defined on a scale $\{S, \leq\}$, with S a set and \leq a total ordering of S – we assume that S and the ordering relation \leq are determined by the adjective that *how* combines with. Antonyms such as *fast* and *slow* have scales based on the same set but are associated with opposite ordering relations.)

In order to solve the above-mentioned problems, we postulate that the degree-question operator *how* carries an additional presupposition, which we express by means of the following syntacategorematic rule:

- (24) (23) presupposes that there is an interval I defined on scale S such that:
 a. $\phi(I)$ is true, and
 b. I 's lower bound m is not 0 and for any m' such that $m \leq m'$, m' is in I .
 In the case of a scale such as the one associated with *fast*, which has no upper bound, condition b. amounts to a claim that I is of the form $[m, +\infty[$ or $]m, +\infty[$.

3.2.2. Consequences of the proposal

Solving Problem #1.

- (25) How fast are we required to drive on the highway?

Given the assumption made in subsection 3.2.1, (25) presupposes that for some interval I of the form $[m, +\infty[$ or $]m, +\infty[$, with $m \neq 0$, we are required to drive at a speed contained in I . In other words, (25) presupposes that there is a minimal permitted speed, which was the desired result. Note that this presupposition does not entail that there is no maximal speed. For if the rules say that our speed must be between, say 50mph and 75mph, it follows, among other things, that our speed must be included in $[50mph, +\infty[$, and therefore the presupposition is satisfied.

Solving Problem #2.

- (26) How fast are we not allowed to drive on the highway?

(26) is predicted to presuppose that for some interval I of the form $[m, +\infty[$ or $]m, +\infty[$, with $m \neq 0$, we are not allowed to drive at a speed contained in I . This is equivalent to saying that there is a maximal permitted speed, which was, again, the desired result.

How fast vs. How slow.

The presupposition postulated in subsection 3.2.1 ensures that *How fast* and *How slow* are not synonymous, because the interval whose existence is presupposed depends on the directionality of the underlying ordering relation. Thus a question such as *How slow are we required to drive?* is predicted to presuppose that there is an interval I of degrees of slowness of the form $[x, MAX]$ or $]x, MAX]$, where MAX is the maximal degree of slowness (i.e. the null speed), such that our degree of slowness

Even when it is known that there is a maximal speed and no minimal speed, (i-a) is perfectly felicitous, and (i-b) is felicitous as well when it is known that there is a minimal speed and no maximal speed.

must be in I. In other words, it is presupposed that there is a minimal permitted degree of slowness, i.e. a maximal permitted speed.

These additional presuppositions do not lead to wrong results in basic cases. Thus a simple degree question such as *How fast did Jack drive?* will presuppose that for some non-null speed m , Jack's speed was above m , i.e. that Jack had a non-null speed. This seems to us to be quite innocent, as it simply amounts to saying that Jack was driving - which such a question appears indeed to presuppose.

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